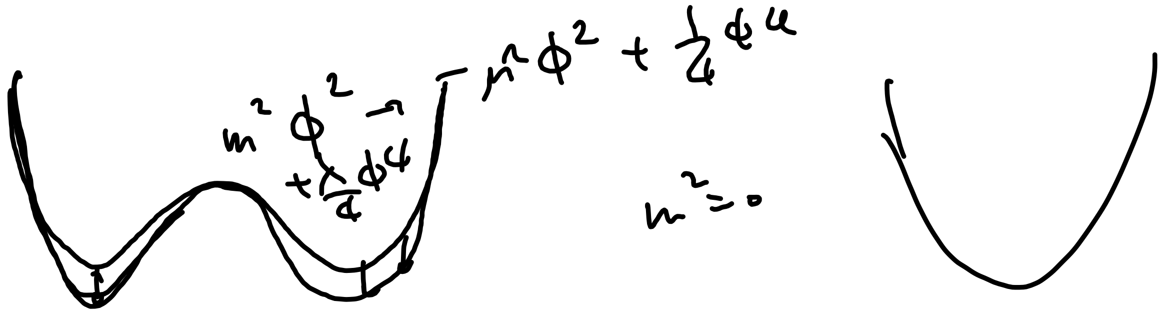
# Chap 12. Renormalization Group.

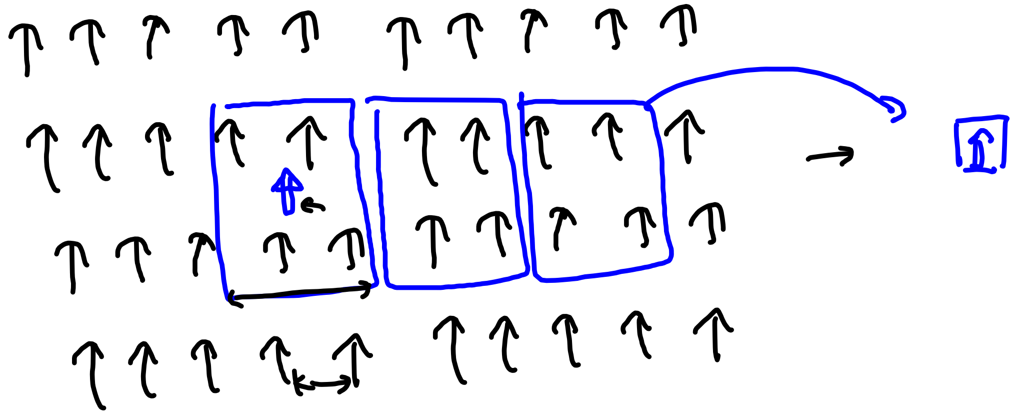
## Final Project

**Coleman-Weinberg Potential**

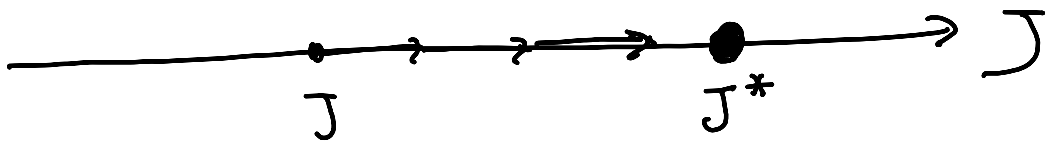



* 6/13에 prensentation 으로 기말고사를 대체함.

# Kadanoff Block spin idea



$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j \longrightarrow -\tilde{J} \sum_{\langle I,J \rangle} \tilde{S}_I \tilde{S}_J$$




(cf) $\lambda(M)$

# K. Wilson's RG

$$Z[J] = \int \mathcal{D}\phi \; e^{i\int(\mathcal{L}+J\phi)} = \underbrace{\int \prod_k d\phi(k)}_{0\le|k|\le\Lambda} \; e^{i\int(\mathcal{L}+J\phi)}$$

$$\phi(x) \leftrightarrow \phi(k) = \int \frac{d^dx}{(2\pi)^4}\,\phi(x)\,e^{-ik\cdot x}$$

$$\phi(x) = \int \phi(k)\, e^{ikx}\,\frac{d^4k}{(2\pi)^4} = [\phi(k)]_{|k|<b\Lambda} + [\hat{\phi}] \longrightarrow \phi + \hat{\phi}$$


$$\hat{\phi}(k) = \begin{cases} \phi(k), & b\Lambda \le |k| < \Lambda \\ 0 & \text{otherwise} \end{cases}$$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4$$

$\phi + \hat{\phi}$

$(k \le b\Lambda)$

$$Z = \int \mathcal{D}\phi \underbrace{\int \mathcal{D}\hat{\phi}}\; e^{i\int \overbrace{\mathcal{L}[\phi+\hat{\phi}]}}$$

$$\mathcal{L}[\phi] + \frac{1}{2}(\partial_\mu\hat{\phi})^2 + \frac{1}{2}m^2\hat{\phi}^2 + \lambda\left(\frac{1}{6}\phi^3\hat{\phi} + \frac{1}{4}\phi^2\hat{\phi}^2 + \frac{1}{6}\phi\hat{\phi}^3 + \frac{\hat{\phi}^4}{4!}\right)$$

$$+ \cancel{\phi\hat{\phi}} + \cancel{\partial_\mu\phi\,\hat{\phi}}$$

$$\int d^4x\, \partial_\mu\phi(x)\,\hat{\phi}(x) = \int d^4k\, k\, \underbrace{\phi(k)}\, \underbrace{\hat{\phi}(-k)} = 0$$

$$\mathcal{L}_0 = \int d^d x \; \frac{1}{2} (\partial_\mu \underset{\uparrow}{\hat{\phi}})^2 \quad \rightarrow \quad \underbrace{\int \frac{d^d k'}{(2\pi)^d} \underbrace{\hat{\phi}(-k')}_{= \hat{\phi}(k')^*} k'^2 \; \hat{\phi}(k')}_{b\Lambda \le |k| < \Lambda}$$

real $\qquad\qquad$ complex

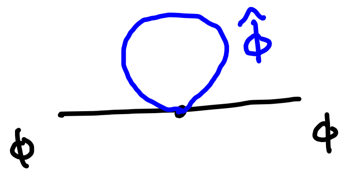
$$\overbrace{\hat{\phi}(k)\,\hat{\phi}(p)} = \frac{\int \mathcal{D}\hat{\phi}\; \hat{\phi}(k)\,\hat{\phi}(p)\; e^{-\boxed{\int \mathcal{L}_0}}}{\int \mathcal{D}\hat{\phi}\; e^{-\int \mathcal{L}_0}} = \frac{1}{k^2} (2\pi)^d \, \delta^{(d)}(k+p)\, \Theta(k)$$

$$\Theta(k) = \begin{cases} 1, & b\Lambda < |k| < \Lambda \\ 0, & \text{아니면} \end{cases}$$

after Taylor expanding interaction terms,

(ex)

$$-\int d^d x \; \frac{\lambda}{4} \underbrace{\phi^2 \hat{\phi}}\hat{\phi} \Rightarrow \int \mathcal{D}\hat{\phi} \left[ -\int d^d x \; \frac{\lambda}{4} \phi^2 \overbrace{\hat{\phi}\,\hat{\phi}} \right] e^{-\int \frac{d^d k'}{(2\pi)^d} \hat{\phi}(-k') k'^2 \hat{\phi}(k')}$$

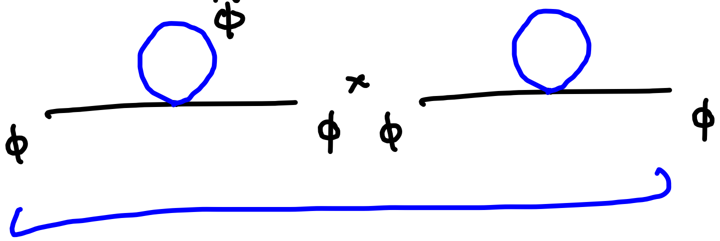



$$= -\frac{1}{2} \int \mu \; \phi(k_1)\, \phi(-k_1) \frac{d^d k_1}{(2\pi)^d} \xrightarrow{\text{exponentiate}} e^{-\int d^d x \, \frac{\mu}{2} \phi^2}$$

$$\mu = \frac{\lambda}{2} \underbrace{\int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2}}_{\int_{b\Lambda}^{\Lambda} \frac{k^{d-1}\, dk}{(2\pi)^d} \frac{1}{k^2}} = \frac{\lambda}{2} \cdot \frac{1}{(4\pi)^{d/2}} \underbrace{\int_{b\Lambda}^{\Lambda} k^{d-3}\, dk}_{\frac{\Lambda^{d-2} - (b\Lambda)^{d-2}}{d-2}}$$

$$-\underbrace{\frac{1}{4!}}\int d^dx\,\zeta\,\phi^4$$

$\uparrow$

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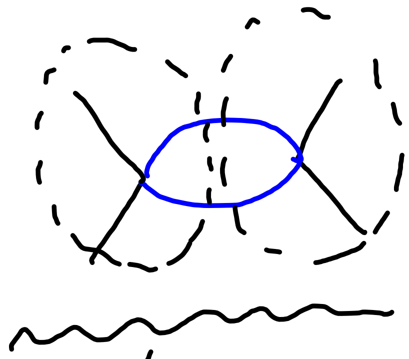
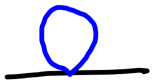


$e$ 

$$\zeta = -4!\,\frac{2}{2!}\left(\frac{\lambda}{4}\right)^2\int \underbrace{\frac{d^dk}{(2\pi)^d}\left(\frac{1}{k^2}\right)^2}_{\int_{b\Lambda}^{\Lambda} k^{d-5}\,dk}$$

$$= -\frac{3\lambda^2}{(4\pi)^{d/2}\,\Gamma\left(\frac{d}{2}\right)}\cdot\frac{\Lambda^{d-4}-(b\Lambda)^{d-4}}{d-4}$$

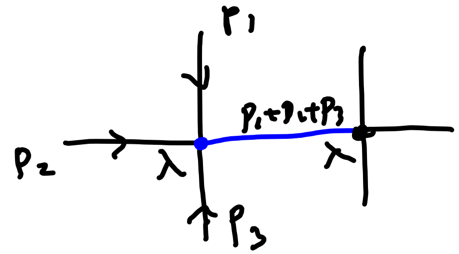
$d-4=\varepsilon$

$d\to 4$ ;

$$\zeta = \cdots\cdot\frac{1}{\varepsilon}\left(\underbrace{e^{\varepsilon\log\Lambda}-e^{\varepsilon\log(b\Lambda)}}_{\cancel{1}+\underbrace{\varepsilon\ln\Lambda}+\cdots-(\cancel{1}+\underbrace{\varepsilon\ln b\Lambda}+\cdots)}\right)$$

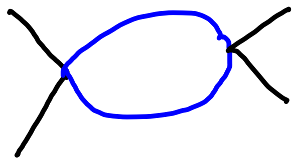
$$= \cdots\cdot\log\frac{\cancel{\Lambda}}{b\cancel{\Lambda}} = \underline{-\frac{3}{16\pi^2}\lambda^2\log\frac{1}{b}}$$

$\phi^6$




$$\sim \frac{\lambda^2}{(p_1+p_2+p_3)^2}$$

$\phi^2(\underset{\sim}{\partial_\mu}\phi)^2$



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$$\mathcal{L}_{eff} = \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4}_{\phi(k)\quad |k|\leq b\Lambda} + \left(\text{connected diagrams}\right.$$

$$\left.\mu\phi^2 + \zeta\phi^4 + \underline{\overset{D}{\alpha}\phi^6} + \rho\phi^2\underline{(\partial_\mu\phi)^2} + \cdots\right)$$

$$k' = \frac{k}{b} \to |k'| \leq \Lambda, \quad \Longleftrightarrow \quad x' = bx \qquad (b<1)$$

$$\int d^dx\,\mathcal{L}_{eff} = \int d^dx'\, b^{-d}\Big[\underbrace{\frac{1}{2}(1+\Delta Z)(\partial_\mu\phi)^2}_{\frac{1}{2}(\partial_\mu\phi')^2} + \underbrace{\frac{1}{2}(m^2+\Delta m^2)\phi^2}_{\frac{1}{2}m'^2\phi'^2} + \overbrace{\frac{1}{4}(\lambda+\Delta\lambda)\phi^4}^{\frac{1}{4}\lambda'\phi'^4+\cdots} + \cdots + \cdots\Big]$$

$$\phi' = \left[b^{2-d}(1+\Delta Z)\right]^{1/2}\phi$$

$$m'^2 = (m^2 + \Delta m^2)(1+\Delta Z)^{-1} b^{-2}$$
$$\lambda' = (\lambda + \Delta\lambda)(1+\Delta Z)^{-2} b^{d-4}$$
$$D' = (D + \Delta D)(1+\Delta Z)^{-3} b^{2d-6}$$
$$\vdots$$

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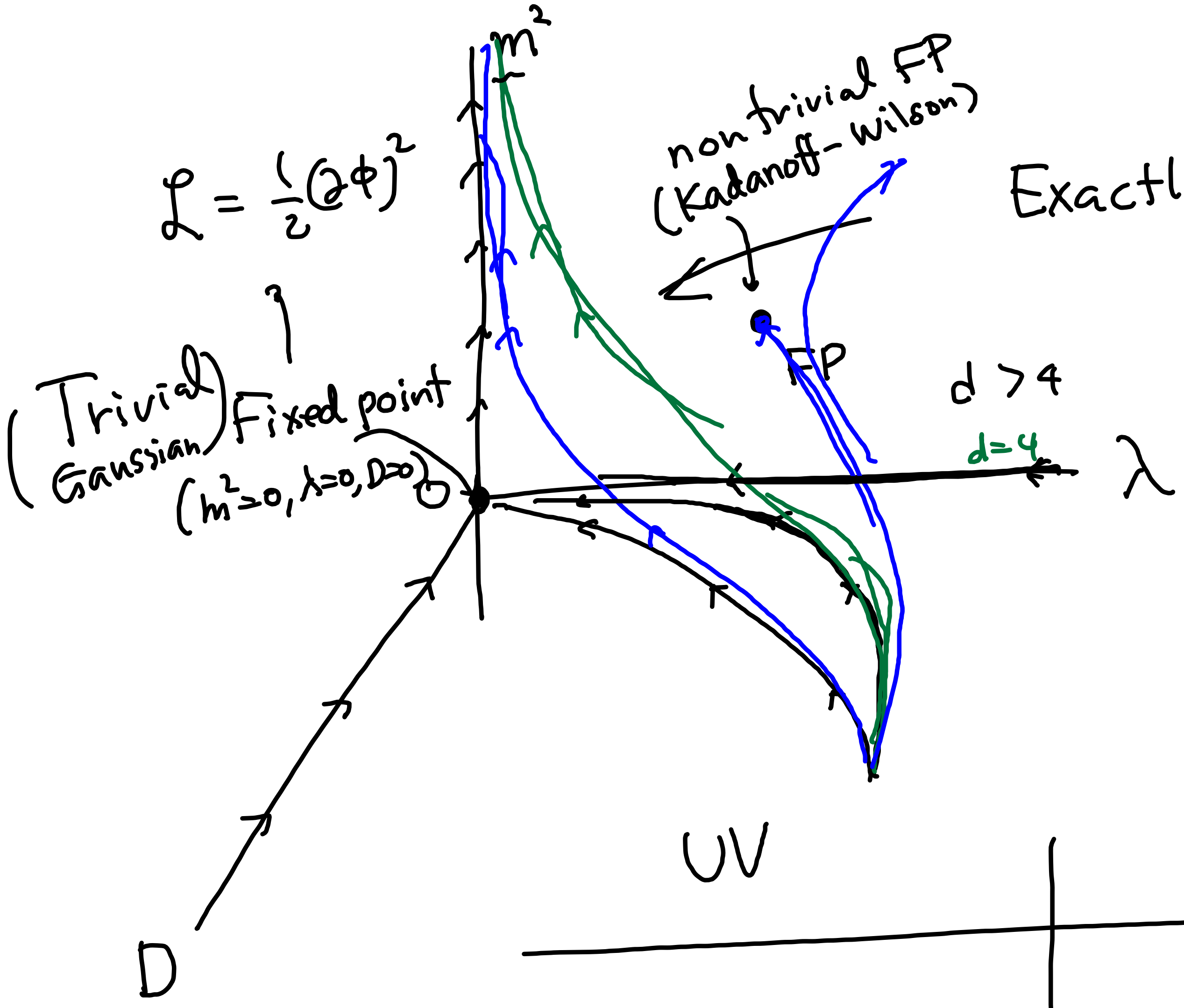
$$\Delta Z = \Delta m^2 = \underset{\sim \lambda^2}{\Delta\lambda} \ll m^2, \lambda, \cdots$$

$b < 1$  $\Lambda \to b\Lambda \to \cdots$

RG 용어

| | | |
|---|---|---|
| $m'^2 = m^2 b^{-2}$ | $b^-$ | relevant (낮은 E에서 중요해짐) |
| $\lambda' = \lambda\, b^{d-4}$ | $b^0$ | marginal |
| $D' = D\, b^{2d-6}$ | $b^+$ | irrelevant ( 〃 무시가능) |

# Wilson's parameter space



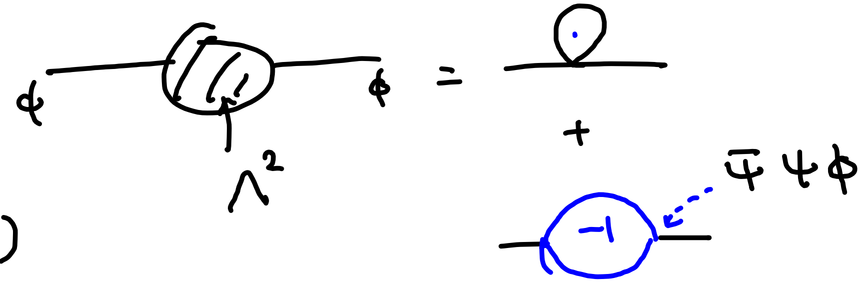

Exactly Solvable QFT

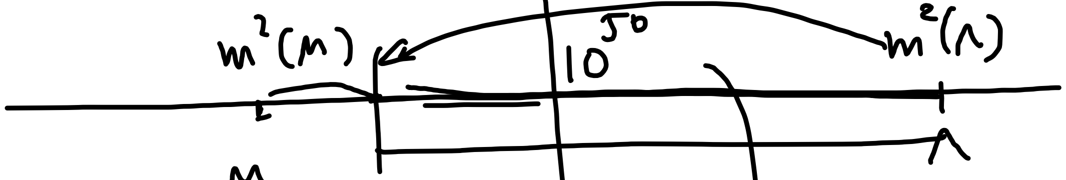
exact RG trajectory possible.

| UV | IR |
|---|---|
| | Relevant parameters only |

$m^2$ : relevant

$$m^2(M) = m^2(\Lambda) + \overbrace{\left(\frac{\Lambda}{M}\right)^2}^{\delta m^2}$$




Higgs 100 GeV

$\uparrow$

$m_H = 125$ GeV

$\uparrow$

$M_{pl} \sim 10^{30}$ GeV

$$\overbrace{1000000\cdots}^{25}$$

$$m^2(\Lambda) = -\left( \quad '' \quad - \underline{125} \right)$$

## <u>Naturalness problem</u>

$\Rightarrow$ scalar field can not be fundamental.

technicolor

$\phi$ = meson $(q\bar{q})$

supersymmetry

$\phi, \psi(x)$

superspace

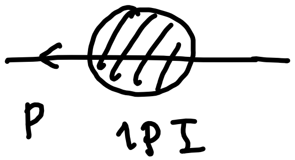
$$\Phi(\underbrace{x, \theta}) = \phi(x) + \theta\psi(x)$$

# 12.2. Callan-Symanzik Eq. (RG eq.)

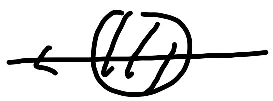
<u>neglect irrelevant</u>

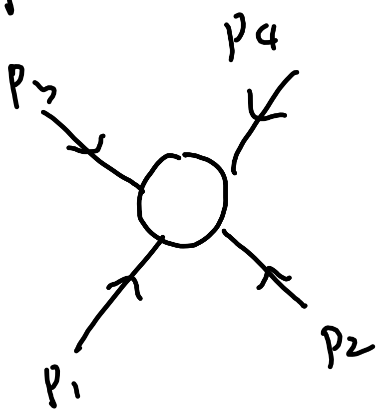
scale (M) should be fixed;

$\phi^4$ theory の renomalization scale M




$= 0$ if $p^2 = -M^2$

$\frac{d}{dp^2}$  $= 0$ if "




$= -i\lambda$ if $\underset{(p_1+p_3)^2}{\underset{\uparrow}{t}} = \underset{(p_1+p_2)^2}{\underset{\uparrow}{s}} = \underset{(p_1+p_4)^2}{\underset{\uparrow}{u}} = -M^2$

Consider a Correlation function

bare ; $\langle \Omega | T(\phi_0(x_1) \cdots \phi_0(x_n)) | \Omega \rangle$

$$\phi = Z^{-\frac{1}{2}} \phi_0$$

$$\langle \Omega | T(\phi(x_1) \cdots \phi(x_n)) | \Omega \rangle = G^{(n)}(x_1 \cdots x_n)[\lambda, M]$$

$$= \underset{\sim}{Z}^{-\frac{n}{2}} \langle \Omega | T(\phi_0(x_1) \cdots \phi_0(x_n)) | \Omega \rangle$$

$M \to M + \delta M$

$\lambda \to \lambda + \delta\lambda$

$\phi \to \underbrace{(1 + \underset{\sim}{\delta\eta})}_{\parallel \; Z^{-\frac{1}{2}}} \phi$

$Z^{-\frac{n}{2}} = (1 + \delta\eta)^n = 1 + \underline{\underline{n\,\delta\eta}}$

$$\Rightarrow \quad \delta G^{(n)} = \frac{\partial G^{(n)}}{\partial M} \delta M + \frac{\delta G^{(n)}}{\delta \lambda} \delta\lambda$$

$$= +n\,\delta\eta\, G^{(n)}$$

$$\left( \delta M \frac{\partial}{\partial M} + \delta\lambda \frac{\partial}{\partial \lambda} - n\,\delta\eta \right) G^{(n)} = 0$$

$$\left( \delta M \frac{\partial}{\partial M} + \delta\lambda \frac{\partial}{\partial \lambda} - n\,\delta\eta \right) G^{(n)} = 0$$

$$\beta \equiv \frac{\delta\lambda}{\delta M} \cdot M = \frac{\delta\lambda}{\delta(\log M)} \leftrightarrow$$

$$\gamma \equiv -\frac{\delta\eta}{\delta M} M = -\frac{\delta\eta}{\delta(\log M)}$$

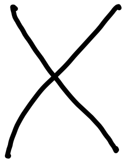
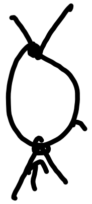
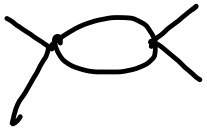
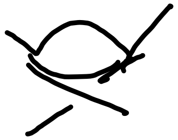
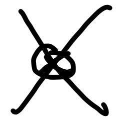
$$\left( \frac{\partial}{\partial M} + \frac{\beta}{M}\frac{\partial}{\partial\lambda} + \frac{\gamma}{M}\cdot n \right) G^{(n)} = 0$$

$$\boxed{\left( M\frac{\partial}{\partial M} + \underbrace{\beta\frac{\partial}{\partial\lambda}} + \gamma\, n \right) G^{(n)} = 0}$$

R-G eq.

dimensionless

$\hookrightarrow \beta(\lambda, \cancel{M})\,,\ \gamma(\lambda)$

$G^{(4)} =$  $+$  $+$  $+$  $+$ 

$-i\delta\lambda$

$$= \left( \underset{\sim}{-i\lambda} + (-i\lambda)^2 \left[ iV(s) + iV(t) + iV(u) \right] - i\delta\lambda \right) \prod_{i=1}^{4} \frac{i}{p_i^2}$$

$$\delta\lambda = (-i\lambda)^2\, 3\, \underset{\sim}{V(-M^2)} = \frac{3\lambda^2}{2(4\pi)^2} \left[ \frac{1}{\varepsilon} - \gamma_E - \underset{\sim}{\log M^2} \right]$$

$$= -\log \frac{M^2}{m^2}$$

$$M \frac{\partial}{\partial M} G^{(4)} = \frac{3i\lambda^2}{(4\pi)^2} \prod_i \frac{i}{p_i^2}$$

$\gamma$ higher order $\qquad +$

$$\beta \frac{\partial}{\partial \lambda} G^{(4)} = \beta \left( -i \prod_{i=1}^{4} \frac{i}{p_i^2} + \text{higher order} \right)$$

$$= 0 \quad \Rightarrow \quad \boxed{\beta(\lambda) = \frac{3\lambda^2}{(4\pi)^2} + \mathcal{O}(\lambda^3)}$$

$$\frac{d\lambda}{d\ln M} = \beta = \frac{3\lambda^2}{16\pi^2} > 0$$

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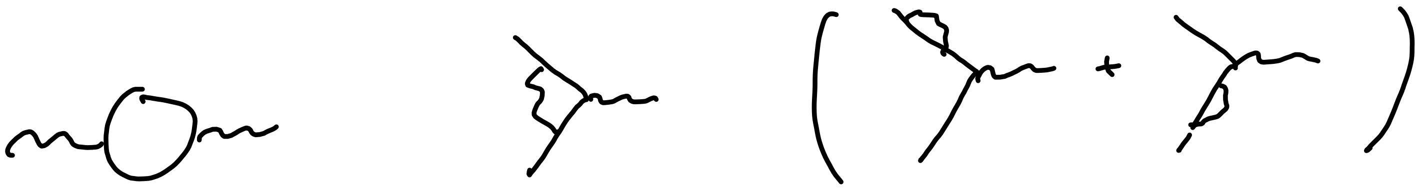
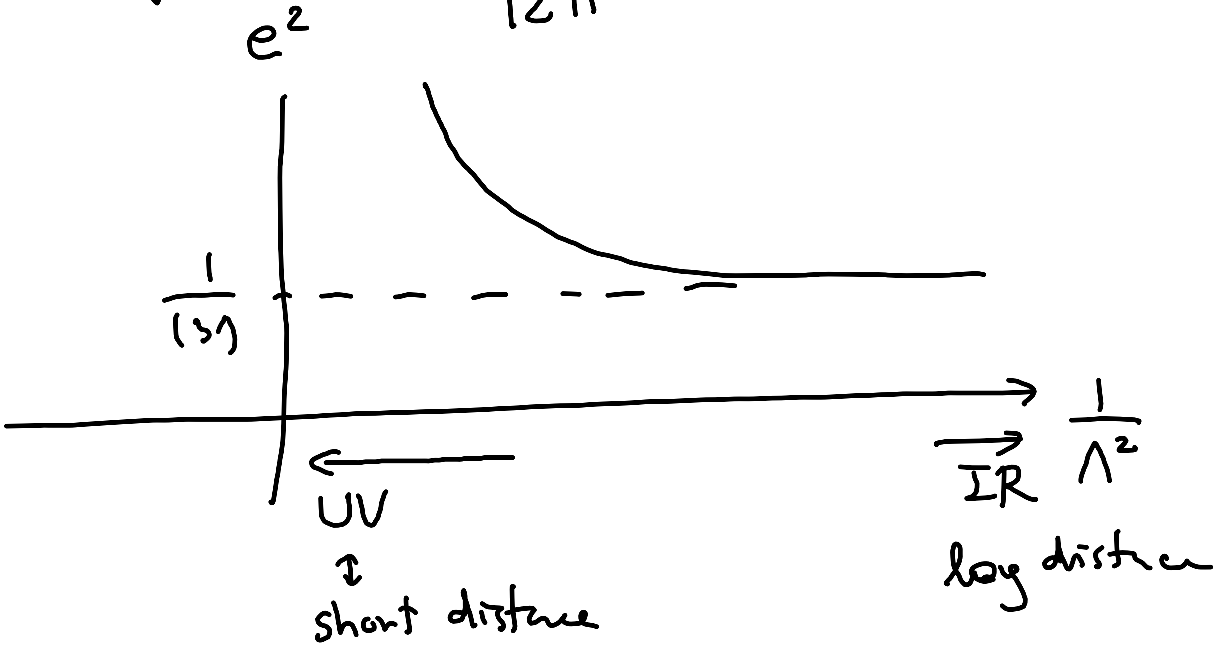
Q.E.D. :



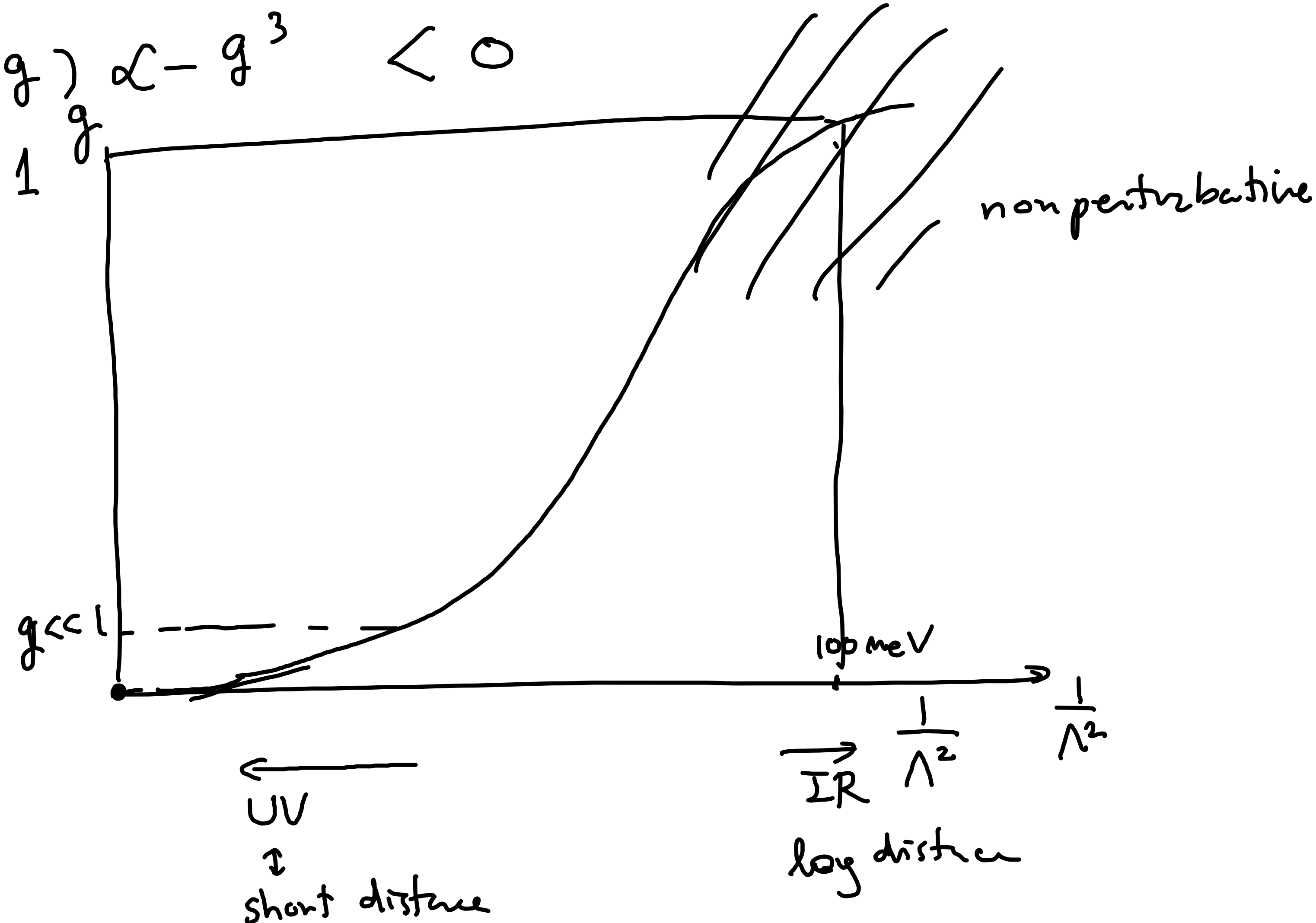
$$\beta(e) = \frac{e^3}{12\pi^2} > 0$$



QCD

Gross, Wilcek, Politzer (2008)

$\beta(g) \propto -g^3 < 0$




Asymptotic freedom

UV

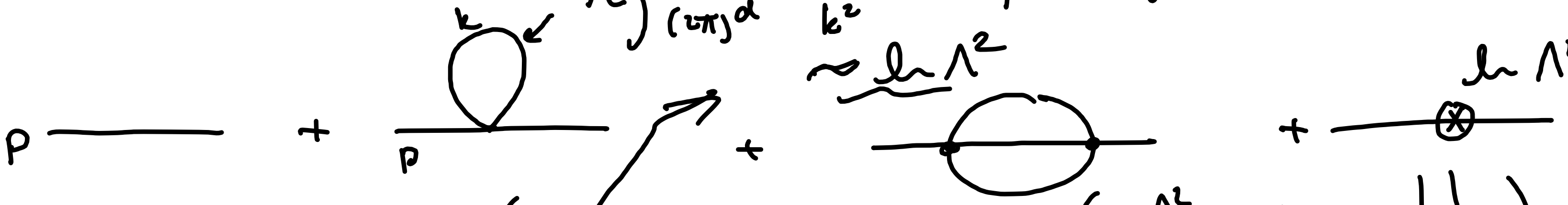
$\frac{1}{2}(\partial_\mu \phi)^2 \ldots$

✓

## 2-pt.

$$\lambda \int \frac{d^dk}{(2\pi)^d} \frac{1}{k^2} \leftarrow p \text{ indep.}$$

$\sim \ln \Lambda^2$ $\qquad$ $\ln \Lambda^2$




$\times$

$\lambda^2 \left( \ln \frac{\Lambda^2}{p^2} + \dots \right)$

$M^2$ independent

$$G^{(2)} = \frac{i}{p^2} + \frac{i}{p^2} \left( A \log \frac{\Lambda^2}{-p^2} + \cdots \right) + \frac{i}{p^2} \, i \cancel{p^2} \delta_Z \frac{i}{\cancel{p^2}}$$

$$\delta_Z = A \ln \frac{\Lambda^2}{M^2} + \cdots$$

$\uparrow$

$\lambda^2$

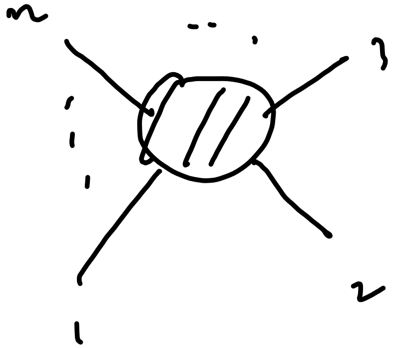
$$= -2 A \ln M + \cdots$$

$$\left( M \frac{\partial}{\partial M} + \beta \frac{\partial}{\partial \lambda} + 2\gamma \right) G^{(2)} = 0$$

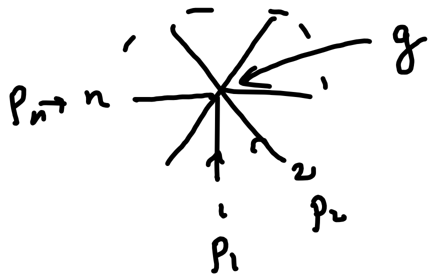
$$M \frac{\partial}{\partial M} G^{(2)} = -\frac{i}{p^2} M \frac{\partial \delta_Z}{\partial M} = -\frac{i}{p^2} \frac{\partial \delta_Z}{\partial \ln M} = \frac{2iA}{p^2}$$

$$\gamma \equiv \frac{1}{2} \frac{\partial \delta_Z}{\partial \ln M} = -A = \mathcal{O}(\lambda^2)$$

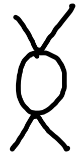
generic vertex

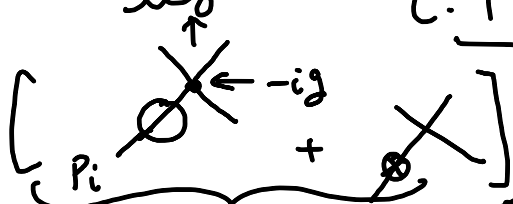



$= G^{(n)}$




= tree + 1PI. 1-loop + external leg + vertex C.T.

(ex) 




$$= (-i)\left(\prod_{i=1}^{n} \frac{i}{p_i^2}\right)\left[ g + B\log\frac{\Lambda^2}{-p^2} + g\sum_i\left(A_i\log\frac{\Lambda^2}{-p_i^2} - \delta_{Z_i}\right) + \delta g\right]$$

C-S Eq.

$$(\text{''})\left[M\frac{\partial}{\partial M}\left(\delta g - g\sum_i \delta_{Z_i}\right) + \beta(g)\frac{\partial}{\partial g}\underbrace{[\quad]}_{\approx 1+\cdots} + g\sum_i \frac{1}{2}M\frac{\partial}{\partial M}\delta_{Z_i}\right] = 0$$

$$\underline{\beta = M\frac{\partial}{\partial M}\left(\frac{1}{2}\sum_i \delta_{Z_i} - \delta g\right)} = -g\left(\sum_i A_i - 2B\right) \qquad \boxed{\delta g = -B\ln\frac{\Lambda^2}{M^2} + \cdots}$$

12.3. $\beta, \gamma$ decided.

## $C\text{-}S \rightarrow G^{(n)}$

$$\left(\underset{\sim}{M}\,\partial_M + \beta\,\partial_\lambda + n\gamma\right) G^{(n)}_{(p)} = 0$$

$\underline{n=2.}$

$$\langle \phi(p)\phi(-p)\rangle = \frac{i}{p^2}\, g\Big(\overbrace{-\underbrace{\frac{p^2}{M^2}}_{\text{dim'less}}}^{-e^{2\overbrace{(\ln p - \ln M)}^{y}}}\Big) = G^{(2)}(p)$$

$$\frac{\partial}{\partial \ln M} G^{(2)}(p) = \frac{i}{p^2}\, g'(-e^{2y})\,(-e^{2y})\,2\overbrace{\frac{\partial y}{\partial \ln M}}^{=-1}$$

$$\frac{\partial}{\partial \ln p} G^{(2)}(p) = \underbrace{\frac{i}{p^2}\, g'(-e^{2y})\,(-e^{2y})\,2\overbrace{\frac{\partial y}{\partial \ln p}}^{1}}_{-\frac{\partial G^{(2)}}{\partial \ln M}} + g(\ )\,\frac{\partial\left(\frac{i}{p^2}\right)}{\partial(\ln p)}$$

$$-2\cdot\frac{i}{p^2} = p\,\frac{\partial}{\partial p}\left(\frac{i}{p^2}\right)$$

$$\left(+\frac{\partial}{\partial \ln p} + 2 - \beta\,\partial_\lambda - 2\gamma\right) G^{(2)} = 0$$

if free theory ; $\beta = \gamma = 0$ $\quad \frac{\partial}{\partial \ln p} G^{(2)} = -2\,G^{(2)} \rightarrow G^{(2)} = \frac{i}{p^2}$

$$\Big(\underbrace{\frac{\partial}{\partial \ln p} - \beta(\lambda)\frac{\partial}{\partial\lambda}}_{\partial_{(p,\lambda)}} + 2 - 2\gamma\Big) G^{(2)} = 0$$

running coupling constant : $\bar\lambda(p,\lambda)$

$$\overbrace{\left(\frac{\partial}{\partial \ln p} - \beta(\lambda)\frac{\partial}{\partial\lambda}\right)}^{\equiv \partial_{(p,\lambda)}} \bar\lambda = 0$$

$$\underbrace{0 = G^{(2)}\left[E^{-1}\frac{\partial E}{\partial \ln p} + (2-2\gamma)\right]}$$

$$G^{(2)}(p) = \tilde{g}(\bar\lambda(p,\lambda))\cdot E(\bar\lambda, p)$$

$$\partial_{(p,\lambda)} G^{(2)}(p) = \left[\partial_{(p,\lambda)}\tilde{g}\right] E + \tilde{g}\underbrace{\left(\partial_{(p,\lambda)} E\right)} = \overbrace{\tilde{g}E}^{G^{(2)}} E^{-1}\frac{\partial E}{\partial \ln p}$$

$$\left[\partial_{(p,\lambda)}\bar\lambda\right]\cdot\frac{d\tilde{g}}{d\bar\lambda} = 0$$

$$\underbrace{\partial_{(p,\lambda)}\bar\lambda}_{=0}\frac{\partial E}{\partial\bar\lambda} + \underbrace{\underbrace{\partial_{(p,\lambda)} p}\frac{\partial E}{\partial p}}_{p\frac{\partial}{\partial p}E = \frac{\partial E}{\partial \ln p}}$$

$$\underbrace{p\frac{\partial}{\partial p}}_{\frac{\partial}{\partial \log p}} p = p$$

$$\frac{dE}{d\ln p} + (2-2\gamma)\,E = 0, \quad \ln E = \int \frac{dE}{E} = \int -(2-2\gamma)\,d\ln p$$

$$\to \quad E = \underbrace{E_0(\bar\lambda)}_{1}\; e^{-\int_M^p (2-2\gamma)\,d\ln p}$$

$$\boxed{G^{(2)}(p,\lambda) = \tilde{g}(\bar\lambda)\; e^{-\int_M^p (2-2\gamma)\,d\ln\frac{p}{M}}}$$

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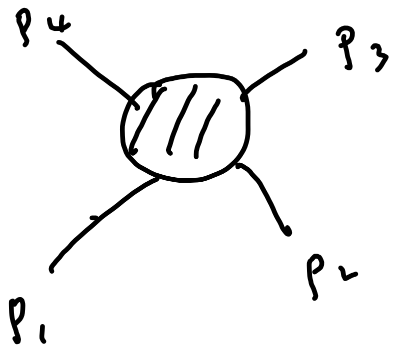



$$= \langle \phi\phi\phi\phi \rangle$$

$$p_i^2 = -P^2 \qquad G^{(4)} = \left(\frac{i}{P^2}\right)^2 \left(-i\lambda + \cdots\right)$$

$$\lambda^2 \log\frac{P^2}{p_i^2} \cdots$$

$$\text{C-S}: \left(P\frac{\partial}{\partial P} - \beta\frac{\partial}{\partial\lambda} + 4 - 4\gamma\right) G^{(4)} = 0$$

$$G^{(4)} = \tilde{G}^{(4)}(\bar\lambda)\; \underbrace{e^{-\int_M^P (4-4\gamma)\,d\ln\frac{P}{M}}}_{\frac{1}{P^4}\cdot e^{\int_M^P 4\gamma\,d\ln\frac{P}{M}}}$$

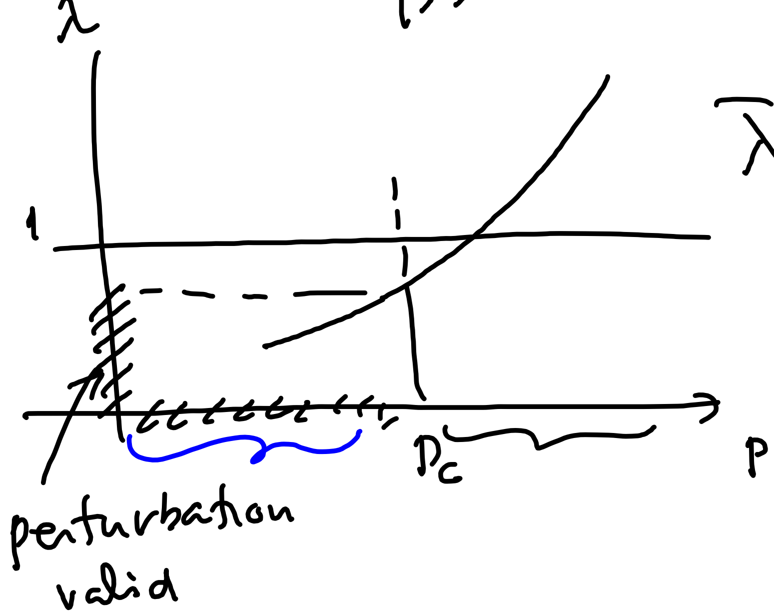
$$\underset{\sim}{\lambda} \ln \frac{p^2}{M^2}$$

$$\left( \frac{\partial}{\partial \ln p} - \beta(\lambda) \frac{\partial}{\partial \lambda} \right) \bar{\lambda} = 0 \quad \rightarrow \quad \frac{\partial \bar{\lambda}}{\partial \ln p} = \beta(\lambda) \frac{\partial \bar{\lambda}}{\partial \lambda}$$

$\bar{\lambda}(M, \lambda) = \lambda$

$$\hookrightarrow \frac{\partial \bar{\lambda}}{\partial \ln \frac{p}{M}} = \beta(\bar{\lambda}) = \frac{3 \bar{\lambda}^2}{16 \pi^2}$$

$\beta > 0$

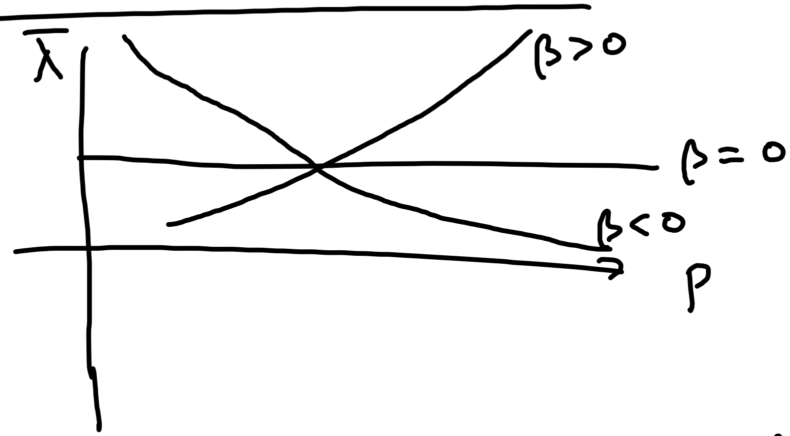



$$\bar{\lambda}(p) = \frac{\lambda}{1 - \frac{3\lambda}{16\pi^2} \ln \frac{p}{M}} \approx \lambda + \delta \lambda$$

$$\delta \lambda = \frac{3 \lambda^2}{16 \pi^2} \ln \frac{p}{M}$$

$$\left( O(N) \rightarrow \frac{(N+8) \lambda^2}{16 \pi^2} \ln \frac{p}{M} \right)$$

$$\frac{d\,\bar{\lambda}}{d\ln\frac{p}{\mu}} = \beta(\bar{\lambda})$$




① $\beta > 0$

QED, $\phi^4$, ...

② $\beta = 0$ ( $\bar{\lambda}$ = const. (no running)

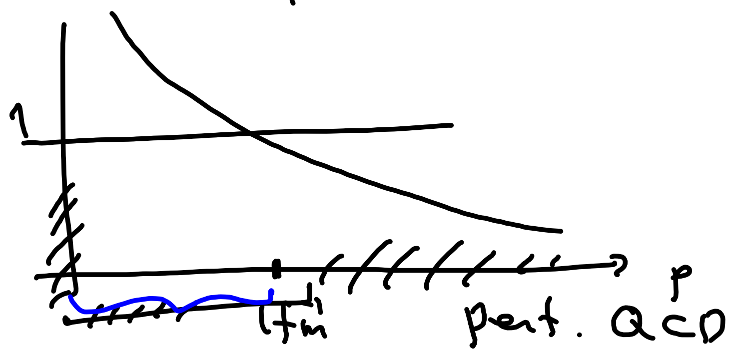
$= \underbrace{\text{scale-invariant} + \text{Poincaré inv.}}_{\text{conformal.}}$ (Lorentz + transl)

AdS/CFT. (d=4 N=4 super Yang Mills)

③ $\beta < 0$ ( Asymptotic free $\bar{\lambda} \to 0$ as $p \to \infty$)

QCD.

nonperturbative QFT.



$$\beta(\lambda = \lambda_*) = 0$$

$$\frac{d\,\bar\lambda}{d\ln p} = \beta(\bar\lambda) = 0$$

$$\bar\lambda = \lambda_*$$

$$\beta(\bar\lambda) = \underset{\overset{\shortparallel}{0}}{\beta(\lambda_*)} + \underbrace{\beta'(\lambda_*)}_{-B}(\bar\lambda - \lambda_*) + \cdots$$

$$-B(\bar\lambda - \lambda_*) = \frac{d\,\overbrace{(\bar\lambda - \lambda_*)}^{\delta\bar\lambda}}{d\log p} \qquad \int \frac{d\,\delta\bar\lambda}{\delta\bar\lambda} = -\int B\,d\ln p$$

$$\shortparallel$$

$$\ln\delta\bar\lambda = -B\ln p$$

$$\bar\lambda - \lambda_* = \delta\bar\lambda(p) = \delta\bar\lambda(M)\,\underbrace{e^{-B\ln\frac{p}{M}}}_{\left(\frac{M}{p}\right)^B}$$

$$\therefore \quad \bar\lambda(p) = \lambda_* + \underbrace{\delta\bar\lambda(M)}_{C}\left(\frac{M}{p}\right)^{B}$$

critical exp.

$$G^{(2)}(p) = \tilde g(\bar\lambda)\, e^{-\int_M^p [2-\gamma(\bar\lambda)]\,d\ln\frac{p}{M}}$$

$$\approx \tilde g(\lambda_*)\,\underbrace{e^{-\int_M^p [2-2\gamma(\lambda_*)]\,d\ln\frac{p}{M}}}_{\left(\frac{M}{p}\right)^{2-2\gamma(\lambda_*)}}$$

$$\therefore \quad G^{(2)}(p) \approx C \cdot \left(\frac{1}{p^2}\right)^{1-\frac{\gamma(\lambda_*)}{\gamma_*}}$$

$$= \langle \phi \, \phi \rangle = \left(\frac{1}{p^2}\right)^{1} \leftarrow \text{free}$$

$$\Rightarrow \quad [\phi] : \underset{\substack{\uparrow \\ \text{naive dim.}}}{1} \longrightarrow \underbrace{1 - \overbrace{\gamma_*}^{\text{anomalous dim.}}}_{\text{quantum dim}}$$

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi)^2}_{4} \quad \phi \to 1$$

pert. "$\lambda^4$"

$(\lambda < 1)$

non pert. of $\gamma(\lambda_*)$

$AdS_5/S^5 \longleftrightarrow$ $N=4$ SYM

string quantum's Energy

$(\lambda \to \infty) \to$ classi:

## Composite operator

$\langle \phi\phi\cdots\phi \rangle$ ;

$\phi^2 \neq \phi\phi$

$\phi^2(x) \leftarrow \phi(x_1)\phi(x_2) -$

$x_1 \to x_2$

$\mathcal{O}: \ \phi^n, \ e^{\alpha\phi}, \ldots$

$$\mathcal{O}_0 = Z_{\mathcal{O}}(M)\,\mathcal{O}_M$$

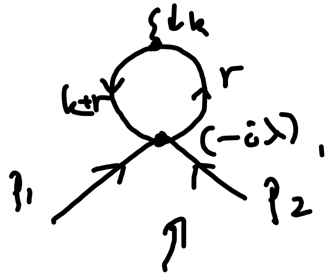
$$G^{(n+1)}(p_1 \cdots p_n, k) = \langle \phi(p_1) \cdots \phi(p_n)\, \mathcal{O}_M(k) \rangle$$

$$= \underbrace{Z(M)^{-\frac{n}{2}}}\ \underbrace{Z_{\mathcal{O}}^{-1}}\ \underbrace{\langle \phi_0(p_1) \cdots \mathcal{O}_0(k) \rangle}$$

$$\left[M\frac{\partial}{\partial M} + \beta\frac{\partial}{\partial \lambda} + n\gamma + \gamma_{\mathcal{O}}\right] G^{(n+1)} = 0$$

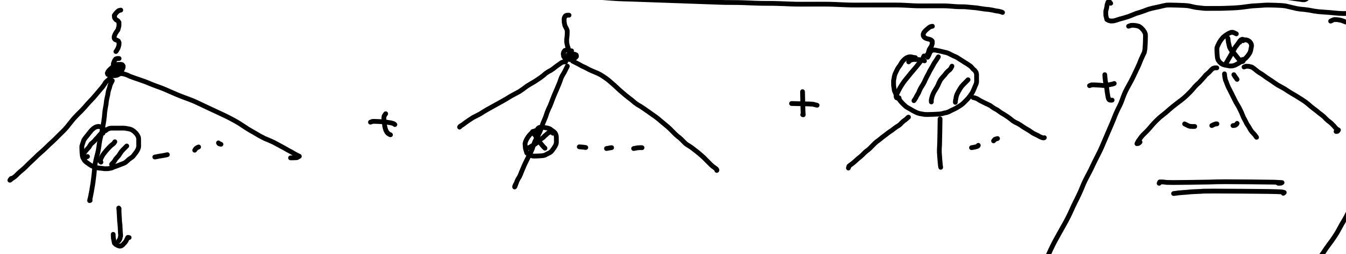
$$\gamma_{\mathcal{O}} = M\frac{\partial}{\partial M}\log Z_{\mathcal{O}}(M)$$

$$\langle \phi_{(p_1)} \phi_{(p_2)} \phi^2(k) \rangle =$$




$$= \frac{i}{p_1^2}\frac{i}{p_2^2}\int \frac{d^4 r}{(2\pi)^d}(-i\lambda)\frac{i}{r^2}\frac{i}{(k+r)^2}$$

$$= \frac{i}{p_1^2}\frac{i}{p_2^2}\left[-\frac{\lambda}{(4\pi)^2}\frac{\Gamma(2-\frac{d}{2})}{\Delta^{2-d/2}}\right]$$



$$\gamma_O = M\frac{\partial}{\partial M}\left(-\delta_O + \frac{n}{2}\delta_Z\right)$$

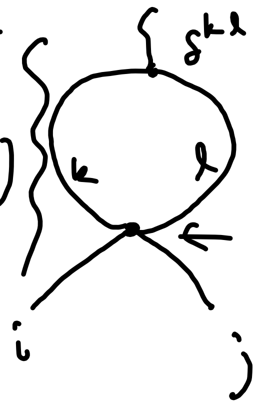
$$\delta_O = +(\quad)$$

$$-\frac{\lambda}{(4\pi)^2}\underbrace{\Gamma(2-\tfrac{d}{2})\, e^{-(2-\frac{d}{2})\ln\Delta}}_{\left(\frac{1}{\varepsilon} - \ln\Delta^2 - \gamma_E + \dots\right)}$$

$$'' \quad \left(-\frac{1}{\varepsilon} + \gamma_E + \ln M^2\right)$$

(ex) $\gamma_{\phi^2} = \dfrac{\lambda}{16\pi^2}$

$$O(N) \to \frac{\lambda}{16\pi^2}(N+2)$$




$\leftarrow \delta^{ik}\delta^{jl}$

$$\underbrace{+\ \text{cycl}(i \rightleftarrows)\ \delta^{kl}\delta^{ij}}_{\delta^{ij} + \delta^{ij} + N\delta^{ij} = (N+2)\delta^{ij}}$$

## $d < 4$

$d$ $\quad \underbrace{\frac{1}{4}(\lambda M^{4-d})}_{\lambda_d} \cdot \phi^4$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2} m^2 \phi^2 - \frac{\lambda_d}{4}\phi^4$$

$[\phi] = \frac{d-2}{2}$ $\rightarrow$ $[\lambda_d] + 4 \cdot \frac{d-2}{2} = d$

$[\lambda_d] = 4 - d$

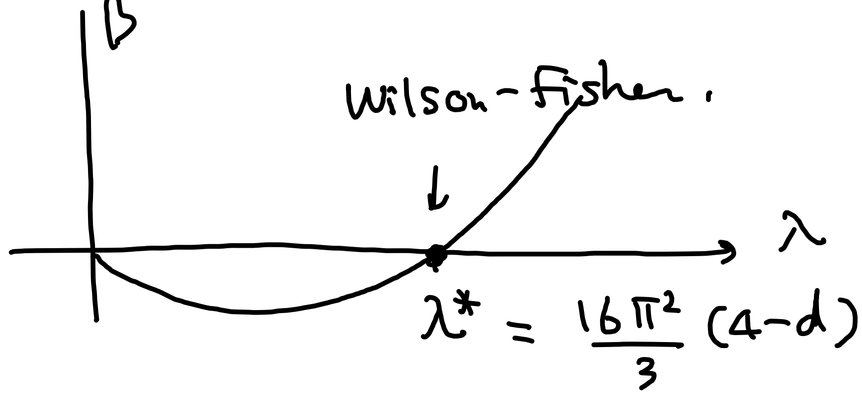
$\therefore \lambda_d = \lambda \cdot M^{4-d}$

dimless

$e^{(4-d)\ln M}$

$$\beta^{(4)} = \frac{\partial(\lambda M^{4-d})}{\partial \ln M} = \underbrace{\frac{\partial \lambda}{\partial \ln M}}_{\beta} + (4-d)\underbrace{\lambda M^{4-d}}_{\lambda_d}$$

$$\beta = (d-4)\lambda + \underbrace{\beta^{(4)}}_{\frac{3\lambda^2}{16\pi^2}} = (d-4)\lambda + \frac{3\lambda^2}{16\pi^2}$$



$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\underset{\substack{\uparrow \\ \text{dim'less}}}{\underline{\underline{g_m}}} M^2 \phi^2$$

$$\bar{g}_m = g_m \left(\frac{M}{\mu}\right)^{2-2\gamma_*} \qquad \gamma_* \leftarrow \gamma_{\phi_2}(\lambda_*)$$

$$\frac{\lambda_*}{\cancel{16\pi^2}} = \frac{\cancel{16\pi^2}}{3}(4-d)$$

$$\xi \sim g_m^{-\nu} \quad \rightarrow \quad \nu = \frac{1}{2-2\gamma_*}$$

$$\nu^{-1} = 2 - 2\left(\frac{4-d}{3}\right)$$

---

$O(N)$:

$$\beta = (d-4)\lambda + \frac{\lambda^2 (N+8)}{16\pi^2} \rightarrow \lambda_* = \frac{16\pi^2}{(N+8)}(4-d)$$

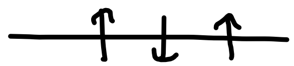
$$\gamma_{\phi^2}(\lambda_*) = \frac{\lambda_*}{16\pi^2}(N+2) = \frac{N+2}{N+8}(4-d)$$

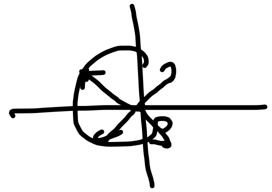
$$\nu^{-1} = 2 - \frac{N+2}{N+8}(4-d)$$

$O(N)$ scalar QFT = magnetic in $d = 3$

| $\downarrow$ | exp |
| --- | --- |
| 0.6 | 0.64 |
| 0.63 | 0.67 |
| 0.65 | 0.71 |

$\xi \sim |T - T_c|^{-\nu}$

N=1; z-축 

N=2; 

N=3; 3차 magnetic

$$\nu^{-1} = 2 - \frac{N+2}{N+8}\underbrace{(4 - \underset{3}{\underset{\uparrow}{d}})}_{1} = 2 - \frac{1}{3} = \frac{5}{3} \to \nu = \frac{3}{5}$$

---

$$G^{(n)} = \tilde{G}(\bar\lambda)\, e^{-\int n(1-\gamma)\, d\ln\frac{p}{M}} \xrightarrow[\lambda = \lambda_*]{} \left(\frac{M}{p}\right)^{\gamma_*}$$

$\tilde{G}(\bar\lambda)$ → pert.

$\tilde{G}_4 = -i\bar\lambda$ ← $\lambda \ln\frac{p}{\mu}$

$G^{(4)} = -i\lambda$

$\lambda = \lambda_*$ ↑ $\beta = 0$

CFT

$\beta = 0$

→ Conformal group ← $d=2$ ∞ dim

6/10 까지 : 6/13 (발표 : 목)

Final project by e-mail.