

On the absence of reflection in AdS_4/CFT_3

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ABSTRACT: A noteworthy feature of the S -matrix which has been proposed for AdS_4/CFT_3 is that the scattering of an A -particle (“soliton”) with a B -particle (“antisoliton”) is reflectionless. We argue, following Zamolodchikov, that the absence of reflection is a result of the existence of certain local conserved charges which act differently on the two types of particles.

KEYWORDS: AdS-CFT Correspondence, Exact S-Matrix, Integrable Field Theories, Bethe Ansatz

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1 Introduction

The AdS_4/CFT_3 correspondence [1] relates type IIA superstring theory on $AdS_4 \times CP^3$ and planar three-dimensional $\mathcal{N} = 6$ superconformal Chern-Simons gauge theory. Evidence for integrability has been found for both the string theory [2]–[10] and the gauge theory [11]–[17]. Based on the symmetries and the spectrum of elementary excitations [11, 18–20], an all-loop S -matrix has been proposed [21] (paralleling the one [22]–[29] for AdS_5/CFT_4) which leads to the all-loop asymptotic Bethe ansatz equations proposed in [30].

The elementary excitations consist of so-called A -particles and B -particles. Since these particles are related by CP symmetry, they can be regarded as “solitons” and “antisolitons,” respectively [9]. A noteworthy feature of the proposed AdS_4/CFT_3 S -matrix is that the scattering of an A -particle with a B -particle is reflectionless. This property has been explicitly verified both at weak coupling [31] and at strong coupling [9]. However, an explanation for this property has been missing.

A possible clue comes from the observation that various integrable relativistic (1+1)-dimensional quantum field theories share this feature of reflectionless scattering. One example is the thermal perturbation of the 3-state Potts model [32], with an S -matrix which is related to the A_2 affine Toda field theory (ATFT). Zamolodchikov [33] showed that the absence of reflection in this model is a result of the existence of higher spin (greater than 1) local integrals of motion which act differently on the particle and on the antiparticle. Similar examples are provided by the thermal perturbation of the tricritical 3-state Potts model, which is related to the E_6 ATFT [34]; and the D_4 ATFT [35].

In this note, we propose an analogous explanation for the absence of reflection in AdS_4/CFT_3 . In section 2, we identify certain local conserved charges, and argue that they imply the reflectionless property. In section 3 we reach the same conclusion from consideration of the algebraic curve. Finally, in section 4 we briefly discuss our results.

2 Local charges and the reflectionless property

The two-loop dilatation operator of $\mathcal{N} = 6$ superconformal Chern-Simons gauge theory has been studied in [11]–[17]. For the $su(4)$ sector [11, 12], there are two associated commuting transfer matrices $\tau(u), \bar{\tau}(u)$. That is, these transfer matrices obey

$$[\tau(u), \tau(v)] = 0, \quad [\tau(u), \bar{\tau}(v)] = 0, \quad [\bar{\tau}(u), \bar{\tau}(v)] = 0, \quad (2.1)$$

for arbitrary values of the spectral parameters u and v . Hence, there are two sets of local charges,

$$Q_n = \frac{d^{n-1}}{du^{n-1}} \ln \tau(u) \Big|_{u=0}, \quad \bar{Q}_n = \frac{d^{n-1}}{du^{n-1}} \ln \bar{\tau}(u) \Big|_{u=0}, \quad n = 1, 2, \dots \quad (2.2)$$

which are mutually commuting,

$$[Q_n, Q_m] = 0, \quad [Q_n, \bar{Q}_m] = 0, \quad [\bar{Q}_n, \bar{Q}_m] = 0. \quad (2.3)$$

The Hamiltonian (dilatation operator) is proportional to $Q_2 + \bar{Q}_2$, and therefore all the charges are conserved. The eigenvalues of the charges are given by

$$\begin{aligned} Q_n &= \frac{i}{n-1} \sum_{j=1}^{K_4} \left(\frac{1}{(u_{4,k} + \frac{i}{2})^{n-1}} - \frac{1}{(u_{4,k} - \frac{i}{2})^{n-1}} \right), \\ \bar{Q}_n &= \frac{i}{n-1} \sum_{j=1}^{K_{\bar{4}}} \left(\frac{1}{(u_{\bar{4},k} + \frac{i}{2})^{n-1}} - \frac{1}{(u_{\bar{4},k} - \frac{i}{2})^{n-1}} \right), \end{aligned} \quad (2.4)$$

where $u_{4,k}$ and $u_{\bar{4},k}$ are the ‘‘momentum-carrying’’ Bethe roots. The same is true for the $osp(4|2)$ sector, as well as for the full two-loop model [14]. We shall assume that these charges lift to the all-loop asymptotic model, as in section 7 of [36],¹ to yield conserved charges with eigenvalues

$$\begin{aligned} Q_n &= \frac{i}{n-1} \sum_{j=1}^{K_4} \left(\frac{1}{(x_{4,k}^+)^{n-1}} - \frac{1}{(x_{4,k}^-)^{n-1}} \right), \\ \bar{Q}_n &= \frac{i}{n-1} \sum_{j=1}^{K_{\bar{4}}} \left(\frac{1}{(x_{\bar{4},k}^+)^{n-1}} - \frac{1}{(x_{\bar{4},k}^-)^{n-1}} \right), \end{aligned} \quad (2.5)$$

where

$$\begin{aligned} x + \frac{1}{x} &= \frac{u}{h(\lambda)}, \\ x^\pm + \frac{1}{x^\pm} &= \frac{u \pm \frac{i}{2}}{h(\lambda)}. \end{aligned} \quad (2.6)$$

¹We are grateful to B. Zwiebel for correspondence on this point.

The still-unknown function h of the 't Hooft parameter λ must satisfy $h(\lambda) \simeq \lambda$ for $\lambda \ll 1$, and $h(\lambda) \simeq \sqrt{\lambda/2}$ for $\lambda \gg 1$. In proposing the existence of a set of charges with eigenvalues (2.5) in the full theory, we are making a stronger claim for the conserved charges than [30], where it was claimed that the spectrum of all conserved charges is given by a set of numbers $\{\mathcal{Q}_n\}$, where $\mathcal{Q}_n = Q_n + \bar{Q}_n$ in our notations. Purely at the level of the spectrum, one might argue that the value of Q_n for a state with K_4 u_4 roots and $K_{\bar{4}}$ $u_{\bar{4}}$ roots could be replicated as the value of \mathcal{Q}_n for a state with the same number of u_4 roots and no $u_{\bar{4}}$ roots, but this is not the case: the two types of roots interact in the full set of Bethe ansatz equations, and hence affect each other's positions. Though somewhat subtle, this difference from [30] is crucial to our understanding of the reflectionless property, because it allows us to distinguish between the A and B particles.

We now introduce (as in [21]) Zamolodchikov-Faddeev operators $A_i^\dagger(p)$ and $B_i^\dagger(p)$ ($i = 1, \dots, 4$) corresponding to A -particles and B -particles, respectively. These particles are associated with the two types of momentum-carrying Bethe roots u_4 and $u_{\bar{4}}$, respectively. (The momenta are given by $e^{ip} = \frac{x_4^+}{x_4^-}$ and $e^{ip} = \frac{x_{\bar{4}}^+}{x_{\bar{4}}^-}$, respectively.) It follows from (2.5) that

$$Q_n A_i^\dagger(p) |0\rangle = q_n(p) A_i^\dagger(p) |0\rangle, \quad q_n(p) = \frac{i}{n-1} \left(\frac{1}{(x_4^+)^{n-1}} - \frac{1}{(x_4^-)^{n-1}} \right),$$

$$Q_n B_i^\dagger(p) |0\rangle = 0, \tag{2.7}$$

and

$$\bar{Q}_n A_i^\dagger(p) |0\rangle = 0,$$

$$\bar{Q}_n B_i^\dagger(p) |0\rangle = \bar{q}_n(p) B_i^\dagger(p) |0\rangle, \quad \bar{q}_n(p) = \frac{i}{n-1} \left(\frac{1}{(x_{\bar{4}}^+)^{n-1}} - \frac{1}{(x_{\bar{4}}^-)^{n-1}} \right). \tag{2.8}$$

The scattering of an A -particle and a B -particle can in principle have both transmission and reflection,

$$A_i^\dagger(p_1) B_j^\dagger(p_2) = S_{ij}^{i'j'}(p_1, p_2) B_{j'}^\dagger(p_2) A_{i'}^\dagger(p_1) + R_{ij}^{i'j'}(p_1, p_2) A_{j'}^\dagger(p_2) B_{i'}^\dagger(p_1), \tag{2.9}$$

where S and R are the transmission and reflection amplitudes, respectively. Acting on (2.9) with the local charge Q_n , and then making use of (2.7), we obtain

$$q_n(p_1) A_i^\dagger(p_1) B_j^\dagger(p_2) |0\rangle = q_n(p_1) S_{ij}^{i'j'}(p_1, p_2) B_{j'}^\dagger(p_2) A_{i'}^\dagger(p_1) |0\rangle + q_n(p_2) R_{ij}^{i'j'}(p_1, p_2) A_{j'}^\dagger(p_2) B_{i'}^\dagger(p_1) |0\rangle. \tag{2.10}$$

Since in general $q_n(p_1)$ is nonzero and $q_n(p_1) \neq q_n(p_2)$, the two equations (2.9) and (2.10) are not compatible unless the reflection amplitudes vanish,

$$R_{ij}^{i'j'}(p_1, p_2) = 0. \tag{2.11}$$

Evidently, the same result can also be obtained by instead acting on (2.9) with the local charges \bar{Q}_n and making use of (2.8). We conclude that the existence of the local charges

Q_n and \bar{Q}_n which act differently on the A -particles and B -particles implies the absence of reflection.²

3 Algebraic curve

The same conclusion can be drawn from a consideration of the algebraic curve [4, 30].³ The A and B particles correspond to the two orientations of certain giant magnon solutions of the string sigma-model [43, 44]. In the language of [45], these correspond to certain “small” giant magnons. Following [43] (see also [4, 46]), we assume that the quasi-momenta are given by⁴

$$\begin{aligned}
 q_1(x) &= \frac{\alpha x}{x^2 - 1}, \\
 q_2(x) &= \frac{\alpha x}{x^2 - 1}, \\
 q_3(x) &= \frac{\alpha x}{x^2 - 1} + G_4(0) - G_4\left(\frac{1}{x}\right) + G_{\bar{4}}(0) - G_{\bar{4}}\left(\frac{1}{x}\right) + G_3(x) - G_3(0) + G_3\left(\frac{1}{x}\right), \\
 q_4(x) &= \frac{\alpha x}{x^2 - 1} + G_4(x) + G_{\bar{4}}(x) - G_3(x) + G_3(0) - G_3\left(\frac{1}{x}\right), \\
 q_5(x) &= G_4(x) - G_4(0) + G_4\left(\frac{1}{x}\right) - G_{\bar{4}}(x) + G_{\bar{4}}(0) - G_{\bar{4}}\left(\frac{1}{x}\right). \tag{3.1}
 \end{aligned}$$

The asymptotic behavior of the quasi-momenta as $x \rightarrow \infty$ determines the various quantum numbers.

In a nutshell, our argument is that the algebraic curves corresponding to

- (i) a particle A of momentum p and a particle B of momentum $-p$, and
- (ii) a particle A of momentum $-p$ and a particle B of momentum p

are different, since the $q_5(x)$ are different. Since the algebraic curve is a classical invariant, there cannot exist a solution with initial asymptotic state (i) and final asymptotic state (ii).

In more detail, let us begin by observing that a single A -particle with momentum p corresponds to

$$G_4(x) = G_{\text{mag}}(x), \quad G_{\bar{4}}(x) = G_3(x) = 0, \tag{3.2}$$

and a single B -particle with momentum p corresponds to

$$G_{\bar{4}}(x) = G_{\text{mag}}(x), \quad G_4(x) = G_3(x) = 0, \tag{3.3}$$

²In this analysis we have neglected the constraint of zero total momentum arising from the cyclicity of the trace [11]. For the case of two particles, this constraint implies the restriction $p_2 = -p_1$. Since there are functions $q_n(p)$ which are not even, the conclusion still holds.

³The AdS_5/CFT_4 algebraic curve was formulated in [37]–[41]; see [42] for a review of the general formalism.

⁴These quasi-momenta should not be confused with the functions $q_n(p)$ used in the previous section.

where [43, 47]

$$G_{\text{mag}}(x) = -i \ln \left(\frac{x - X^+}{x - X^-} \right), \quad (3.4)$$

and $e^{ip} = \frac{X^+}{X^-}$. One can show that A and B particles with momentum $-p$ correspond to (3.2) and (3.3) with $G_{\text{mag}}(x)$ replaced by $\tilde{G}_{\text{mag}}(x)$, respectively, where

$$\tilde{G}_{\text{mag}}(x) = -i \ln \left(\frac{x + X^-}{x + X^+} \right). \quad (3.5)$$

We note that for fundamental (i.e., non-dyonic, $Q = 1$) giant magnons, $q_5(x)$ is of order $1/g$, since $X^\pm = e^{\pm ip/2}[1 + o(1/g)]$. We assume that g is large but finite, so that $q_5(x)$ is nonzero.

Let us consider an initial two-particle configuration consisting of an A -particle with momentum p , and a B -particle with momentum $-p$. As in the ansatz of [46] for the more-complicated case of multi-magnon states in finite volume, we suppose that the quasi-momenta corresponding to this initial state are simply the sums of the quasi-momenta for the two constituent particles. In particular, this means that $q_5(x)$ is

$$q_5(x) \Big|_{\text{initial}} = G_{\text{mag}}(x) - G_{\text{mag}}(0) + G_{\text{mag}}\left(\frac{1}{x}\right) - \tilde{G}_{\text{mag}}(x) + \tilde{G}_{\text{mag}}(0) - \tilde{G}_{\text{mag}}\left(\frac{1}{x}\right). \quad (3.6)$$

Now consider the possible states after the collision has occurred. The “transmitted” configuration again consists of an A -particle with momentum p and a B -particle with momentum $-p$, and so

$$q_5(x) \Big|_{\text{transmitted}} = q_5(x) \Big|_{\text{initial}}. \quad (3.7)$$

The “reflected” configuration consists instead of an A -particle with momentum $-p$ and a B -particle with momentum p ; and therefore

$$\begin{aligned} q_5(x) \Big|_{\text{reflected}} &= \tilde{G}_{\text{mag}}(x) - \tilde{G}_{\text{mag}}(0) + \tilde{G}_{\text{mag}}\left(\frac{1}{x}\right) - G_{\text{mag}}(x) + G_{\text{mag}}(0) - G_{\text{mag}}\left(\frac{1}{x}\right) \\ &= -q_5(x) \Big|_{\text{initial}}. \end{aligned} \quad (3.8)$$

Since in general $q_5(x) \Big|_{\text{initial}}$ is nonzero, it follows that $q_5(x) \Big|_{\text{reflected}} \neq q_5(x) \Big|_{\text{initial}}$; and therefore, reflection is not possible.

The quasi-momenta can be expressed in terms of the scaling limit of the conserved charges (2.4), see e.g. [30, 39]. Hence, the above computation essentially confirms that the set of classical conserved charges is powerful enough to forbid reflection. Note that the quasi-momentum $q_5(x)$, which is of key importance in our argument, behaves like $Q_n - \bar{Q}_n$ and not like $Q_n + \bar{Q}_n$. Indeed, as can be seen from eq. (3.1), $q_5(x)$ is given by the *difference* of quantities involving $G_4(x)$ and $G_{\bar{4}}(x)$, which in turn depend on X_4 and $X_{\bar{4}}$, respectively. In that sense, the classical argument also makes use of the two towers of conserved charges which were essential in the quantum argument of section 2.

It would be valuable to extend these considerations to the full quantum theory, by computing quantum corrections to the local charges that appear in the expansions of the quasi-momenta about $x = \pm 1$, but we will leave this to future work. Even remaining at the classical level, it would be interesting to construct classical solutions corresponding to the scattering of A and B particles, and verify that these solutions do not exhibit reflection.

It may be useful to compare the situation with the more familiar example of the sine-Gordon theory [48]. Classical soliton-antisoliton scattering is also reflectionless in that case. However, unlike the AdS_4/CFT_3 case, reflection is not forbidden by conservation laws. (Hence, in that sense, the absence of reflection in classical sine-Gordon theory is “accidental.”) Indeed, since solitons and antisolitons are only distinguished by a spin-zero (topological) charge, there is no obstruction to a nonzero reflection amplitude appearing in the quantum sine-Gordon theory; and by Gell-Mann’s totalitarian principle that in quantum theory ‘everything that is not forbidden is compulsory’, the quantum sine-Gordon theory does indeed exhibit reflection as well as transmission.⁵ (For an elementary discussion of this point, see [50].) The crucial difference is that in the AdS_4/CFT_3 case there is a momentum-dependent conserved charge which splits apart the A and the B , at all values of the coupling.

4 Discussion

We have argued that the origin of the AdS_4/CFT_3 reflectionless property is the existence of two commuting transfer matrices, and therefore two sets of commuting conserved local charges. This is in contrast to the AdS_5/CFT_4 case, for which there is only one commuting transfer matrix, and therefore only one set of commuting conserved local charges. The argument is a generalization of the one used in the study of purely elastic scattering theories [33–35]. Our discussion of the string sigma-model side of this story has been preliminary, and it would be of particular interest to give a full quantum treatment. In the context of relativistic quantum field theories, local conserved charges often hide intricate structures, such as the Coxeter geometry found in the purely elastic scattering theories (see, for example, [50] and references therein). It would be interesting to see whether similar phenomena also exist in AdS/CFT .

Note added. As we were about to submit this paper to the Arxiv, the paper [51] appeared, which discusses issues related to section 3.

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⁵At a discrete set of values of the coupling, quantum sine-Gordon is again reflectionless [48]. However, at exactly those couplings, it can be argued [49] that the sine-Gordon model picks up an extra conserved charge which *does* split soliton from antisoliton, and so the totalitarian principle survives.

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